Effect of Quark Scatter on Baryogenesis by Electroweak Strings

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Abstract

The amount of baryon asymmetry generated by the electroweak strings at a rough estimate is smaller than that generated by the bubble expansion since the collapsing strings cannot cover the whole volume of the universe. However, the interaction cross section of the strings with particles can be larger than the geometrical area of the order of the string thickness. The scattering cross section of quarks from the electroweak strings is calculated using Dirac equation in the string background. It it proportional to the reverse of the momentum perpendicular to the string axis. Thus the effective interaction area can be enhanced at least ten times so that the suppression compared to the first order phase transition scenario might be also improved.

1 Introduction

The application of the particle physics theory to the early stage of the cosmic evolution provides various interesting ideas which can solve some of the cosmological problems. One of the important examples is the origin of the baryon asymmetry in the universe. The ratio of baryon number density, n_B , to entropy density, s, is estimated as

$$\frac{n_B}{s} = 10^{-10 \sim -9} \tag{1}$$

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by some observations[1, 2]. Although there has been many attempts and intensive study concerning the problem, we do not have the final answer yet.

At present it seems to be probable that the baryon number is generated at the scale of the unification of the electromagnetic and weak interactions ($T \sim 10^2$ GeV), though other possibilities cannot be neglected[1, 3]. As pointed out by Sakharov[4] there are three necessary conditions for baryon asymmetry generation, the existence of baryon number violating process, C and CP symmetry violation, and the deviation from the thermal equilibrium. In the electroweak unified theory, the first condition is satisfied by electroweak anomaly. Sphaleron transitions violate the sum of the baryon number and the lepton number[5]. The second condition is also realized in the electroweak models. The scenario which can solve the baryogenesis problem within the Weinberg-Salam model is still proposed[6]. Note that, however, quite a few people regard the degree of CP invariance violation in the minimal standard model as too small to explain the amplitude of the baryon asymmetry. Various types of model extension are in progress in order to provide sufficient CP violation.

In the traditional electroweak baryogenesis, the out-of-equilibrium condition is achieved by the expansion of the true vacuum bubbles. However, such bubble nucleation occurs only when the order of the phase transition equals one. It seems that the first order electroweak phase transition is difficult to be established in the standard model. Even if the non-standard extension of the theory enables the finite temperature effective potential for the Higgs field to have the energy barrier between the false vacuum and the true vacuum, thermal fluctuations may prevent the bubble nucleation process[7]. The details of the electroweak phase transition is still in progress.

Recently one promising scenario of the electroweak baryogenesis has been proposed[8]. Topological defects are another fruition of particle cosmology[9, 10] and they can be an alternative to bubble walls at the phase transition accompanied by supercooling. Electroweak strings could be produced at the electroweak phase transition[11]. These strings can be the source of the deviation from the thermal equilibrium. Strictly speaking they are not topological defects so that they should collapse into the trivial vacuum configuration. During this process, the boundaries between the false vacuum and the true vacuum move similarly to the walls of nucleated bubbles in the first order phase transition. The interaction of these boundaries with particles realizes the non-equilibrium condition. The electroweak strings themselves have baryon number and can contribute to the baryon asymmetry production[12] or they can introduce baryon number fluctuations through the interaction with the background electromagnetic field[13]. Moreover their effect on the sphaleron transition rate has been discussed[14].

In this paper, we calculate the scattering cross section of quarks from the electroweak strings and discuss the influence upon the baryogenesis scenario. The remainder of the paper is organized as follows. In the next section, the electroweak baryogenesis scenario by the collapses of electroweak strings is briefly reviewed. The quark scattering cross section from the electroweak strings is derived in the section 3. The final section is devoted to

discussion and conclusions.

2 Baryogenesis by Electroweak Strings

The electroweak strings are a kind of non-topological defect. They are topologically unstable so that they have a monopole anti-monopole pair at their ends or their configuration forms loops. Although whether their configuration constitutes a local energy minimum is under investigation[15], it certainly satisfies the equations of the minimal standard electroweak model. If they live out a time then they can contribute to the baryogenesis as the sources of out-of-equilibrium when the decay of the direction along the string axis which looks like the monopole pair annihilation occurs.

In the standard model, the solutions of the Higgs field, Φ , and the spatial components of the Z-boson gauge field, \mathbf{Z} , for a Z-string are written by

$$\Phi = \rho(r) \begin{pmatrix} 0 \\ e^{i\theta} \end{pmatrix} , \qquad (2)$$

$$\mathbf{Z} = \frac{\alpha(r)}{r} \left(-\sin\theta, \cos\theta, 0 \right) , \qquad (3)$$

where r and θ are the radial component and the angular component of the polar coordinate in the plane perpendicular to the string axis and ρ and α are the Nielsen-Olesen solution for the U(1) local string[16]. The equations which these functions obey are

$$\rho'' + \frac{\rho'}{r} - \frac{1}{r^2} (q_{\phi}\alpha - 1)^2 \rho - \lambda (\rho^2 - v^2) \rho = 0 , \qquad (4)$$

$$\alpha'' - \frac{\alpha'}{r} - q_{\phi} \left(q_{\phi} \alpha - 1 \right) \rho^2 = 0 , \qquad (5)$$

where the prime denotes a derivative by r, λ is the self-coupling constant for the Higgs field, v is the expectation value of the Higgs field at zero temperature, and q_{ϕ} is the Z-charge of the Higgs field, i. e.,

$$q_{\phi} = \frac{1}{2} \frac{e}{\sin \theta_w \cos \theta_w} \,, \tag{6}$$

where θ_w is the Weinberg angle. The other gauge fields except **Z** have trivial configurations, *i. e.*, equal zero everywhere.

The similar solution can be obtained for the W-string[17]. However, the energy of the Z-string is lower than that of the W-string. Hence we mainly consider the Z-string hereafter.

The baryon number density produced by the electroweak strings is estimated to be that by the bubble nucleation times the suppression factor [18]

$$\frac{n_B}{s}\Big|_{\text{string}} \sim \frac{n_B}{s}\Big|_{\text{bubble}} \times \frac{V_s}{V} ,$$
 (7)

where V_s is the volume swept out by the collapsing electroweak strings and V is the total volume of the universe. Since V_s can be regarded as the volume occupied by the strings just before their collapse starts, it can be calculated as

$$V_s \simeq \delta_s^2 l_s \frac{V}{\xi_s^3} \,, \tag{8}$$

using the mean separation between strings, ξ_s , the averaged length of the string, l_s , at the string formation epoch and the width scale of the string, δ_s . The length of the string should be comparable to the mean separation, that is, $l_s \sim \xi_s$. Then the suppression factor is represented by

$$\frac{V_s}{V} \sim \left(\frac{\delta_s}{\xi_s}\right)^2 \ . \tag{9}$$

The core radius of the electroweak string is almost the inverse of the Higgs mass scale. If the number density of electroweak strings at the formation epoch is described by the Kibble mechanism[9], then the mean separation of them is equal to the correlation length of the Higgs field, *i. e.*, the inverse Higgs mass scale. In this case, $\xi_s \sim \delta_s$. Thus if the electroweak string distribution is similar to that for the ordinary topological defect, the suppression factor is almost of the order of unity.

In general, $\xi_s \gtrsim \delta_s$. For example, when ξ_s is the particle horizon scale at the electroweak phase transition, V_s/V becomes 10^{-32} . The above estimation (7) is based on the calculation in which the baryon number generation process is active only within the string core. The magnification of the factor has been tried using various particle physics models[19]. Here we take into account the amplification of the quark wave function near the electroweak string.

3 Quark Interaction with Electroweak String

In this section, we derive the scattering cross section of quarks from the electroweak string, σ_s . The calculation method follows that of fermion scattering[20]. In contrast to the calculations in the references [20] where only the particles proceeding perpendicular to the string were considered, we take into account the momentum along the electroweak string axis, which will be proved to be essential to the purpose of finding out the enhancement factor of the baryogenesis by the electroweak strings. Note also that we are interested in the total cross section, not in the helicity conserving one or helicity flipping one. The result shows σ_s is not identical with a naive estimation, *i. e.*, the geometrical cross section, δ_s^2 . It increases as the momentum of the particle becomes nearly parallel with the string.

We express the quark spinor in the standard model as, the SU(2) quark doublet : Ψ_L^a , the right-handed upper quark : ψ_R^{+a} , and the right-handed lower quark : ψ_R^{-a} . Moreover each component of Ψ_L^a is written as ψ_L^{+a} and ψ_L^{-a} , respectively. The superscript, a=1,2,3, represents the family number. Since the following formulae do not depend on

the generation, we omit a from now on. The expressions of Yukawa coupling constants, h^{+a} and h^{-a} , are also simplified to h^+ and h^- . Then Dirac equations for quarks are written by

$$i\gamma^{\mu}D_{\mu}\Psi_{L} = h^{+}\tilde{\Phi}\psi_{R}^{+} + h^{-}\Phi\psi_{R}^{-}, \qquad (10)$$

$$i\gamma^{\mu}D_{\mu}\psi_{R}^{+} = h^{+}\tilde{\Phi}^{\dagger}\Psi_{L} , \qquad (11)$$

$$i\gamma^{\mu}D_{\mu}\psi_{R}^{-} = h^{-}\Phi^{\dagger}\Psi_{L} , \qquad (12)$$

where $\tilde{\Phi} \equiv i\sigma^2\Phi^*$ and D_{μ} is the covariant derivative.

The configurations of the Higgs field and the Z-boson gauge field are arranged corresponding to the solutions of a static straight infinite electroweak string, (2) and (3). Then in the background where an electroweak string exists, Dirac equations for quarks is modified to the formulae such as

$$i\gamma^{\mu}D_{\mu}\psi_{L}^{+} - h^{+}\rho e^{-i\theta}\psi_{R}^{+} = 0$$
, (13)

$$i\gamma^{\mu}D_{\mu}\psi_{L}^{-} - h^{-}\rho e^{i\theta}\psi_{R}^{-} = 0 , \qquad (14)$$

$$i\gamma^{\mu}D_{\mu}\psi_{R}^{+} - h^{+}\rho e^{i\theta}\psi_{L}^{+} = 0$$
, (15)

$$i\gamma^{\mu}D_{\mu}\psi_{R}^{-} - h^{-}\rho e^{-i\theta}\psi_{L}^{-} = 0$$
 (16)

When the γ matrix representation as

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \gamma^k = \begin{pmatrix} 0 & -\sigma^k \\ \sigma^k & 0 \end{pmatrix}, \tag{17}$$

is employed, the left-handed quarks have only two lower components and the right-handed ones have two upper components which we write as

$$\psi_L^+ = \begin{pmatrix} 0 \\ w^+ \end{pmatrix}, \ \psi_L^- = \begin{pmatrix} 0 \\ w^- \end{pmatrix}, \ \psi_R^+ = \begin{pmatrix} u^+ \\ 0 \end{pmatrix}, \ \psi_R^- = \begin{pmatrix} u^- \\ 0 \end{pmatrix}. \tag{18}$$

The equations are simplified to

$$(iD_0 - i\sigma^k D_k) w^+ - h^+ \rho e^{-i\theta} u^+ = 0 ,$$
 (19)

$$(iD_0 + i\sigma^k D_k) u^+ - h^+ \rho e^{i\theta} w^+ = 0 ,$$
 (20)

$$(iD_0 - i\sigma^k D_k) w^- - h^- \rho e^{i\theta} u^- = 0 ,$$
 (21)

$$(iD_0 + i\sigma^k D_k) u^- - h^- \rho e^{-i\theta} w^- = 0.$$
 (22)

For the moment we concentrate on the solutions for the lower quarks, w^- and u^- . The spinors can be decomposed to the eigen states of the total angular momentum around the

string axis as

$$u^{-} = \sum_{j=-\infty}^{+\infty} \begin{pmatrix} u_1^j(r) \\ iu_2^j(r)e^{i\theta} \end{pmatrix} e^{ij\theta + ip_z z - i\omega t} , \qquad (23)$$

$$w^{-} = \sum_{j=-\infty}^{+\infty} \begin{pmatrix} w_1^j(r) \\ iw_2^j(r)e^{i\theta} \end{pmatrix} e^{i(j+1)\theta + ip_z z - i\omega t} , \qquad (24)$$

where (r, θ, z) are components of cylindrical coordinates whose z-axis agrees with the string axis, ω is the total energy of the quark and p_z is the z-component of the quark momentum. Finally the equations for each decomposed spinors are written by

$$\left(\frac{d}{dr} - \frac{j}{r} - 2q_{\phi}q_{R}^{-}\frac{\alpha}{r}\right)u_{1}^{j} + (\omega + p_{z})u_{2}^{j} - h^{-}\rho w_{2}^{j} = 0, \qquad (25)$$

$$\left(\frac{d}{dr} + \frac{j+1}{r} + 2q_{\phi}q_R^{-}\frac{\alpha}{r}\right)u_2^j - (\omega - p_z)u_1^j + h^{-}\rho w_1^j = 0, \qquad (26)$$

$$\left(\frac{d}{dr} - \frac{j+1}{r} - 2q_{\phi}q_{L}^{-}\frac{\alpha}{r}\right)w_{1}^{j} - (\omega - p_{z})w_{2}^{j} + h^{-}\rho u_{2}^{j} = 0,$$
(27)

$$\left(\frac{d}{dr} + \frac{j+2}{r} + 2q_{\phi}q_{R}^{-}\frac{\alpha}{r}\right)w_{2}^{j} + (\omega + p_{z})w_{1}^{j} - h^{-}\rho u_{1}^{j} = 0,$$
(28)

where q_R^- and q_L^- are the Z-charge of quarks :

$$q_R^- = \frac{1}{3}\sin^2\theta_w , \qquad q_L^- = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_w .$$
 (29)

For simplicity we substitute a step-function like formula for $\rho(r)$ and $\alpha(r)$ such as

$$\rho(r) = v
\alpha(r) = \frac{1}{q_{\phi}} \qquad ; \qquad r > \delta_s ,$$
(31)

that is, a trivial configuration inside the string core and the value infinitely far from the string outside the core. Then the differential equations take the form of the Bessel equations and the solutions can be written by the Bessel functions.

Under the condition that the spinors must be regular at the origin, r = 0, the solutions inside the string is expressed by

$$u_1^j = c_j J_j \left(\sqrt{\omega^2 - p_z^2} \ r \right) , \qquad (32)$$

$$u_2^j = c_j D^+ J_{j+1} \left(\sqrt{\omega^2 - p_z^2} \ r \right) ,$$
 (33)

$$w_1^j = d_j J_{j+1} \left(\sqrt{\omega^2 - p_z^2} \ r \right) ,$$
 (34)

$$w_2^j = -d_j D^- J_{j+2} \left(\sqrt{\omega^2 - p_z^2} \ r \right) ,$$
 (35)

where c_i and d_i are arbitrary constants and

$$D^{+} \equiv \sqrt{\frac{\omega - p_z}{\omega + p_z}} , \qquad D^{-} \equiv \sqrt{\frac{\omega + p_z}{\omega - p_z}} . \tag{36}$$

On the other hand, the solutions outside the string core are written by

$$u_1^j = \left(a_j^1 + b_j^1\right) J_{\nu}(R) + \left(a_j^2 + b_j^2\right) J_{-\nu}(R) , \qquad (37)$$

$$u_2^j = \left(a_j^1 A^+ - b_j^1 A^-\right) J_{\nu+1}(R) + \left(-a_j^2 A^+ + b_j^2 A^-\right) J_{-\nu-1}(R) , \qquad (38)$$

$$w_1^j = \left(a_j^1 B^+ + b_j^1 B^-\right) J_\nu + \left(a_j^2 B^+ + b_j^2 B^-\right) J_{-\nu}(R) , \qquad (39)$$

$$w_2^j = \left(a_j^1 A^+ B^+ - b_j^1 A^- B^-\right) J_{\nu+1}(R) + \left(-a_j^2 A^+ B^+ + b_j^2 A^- B^-\right) J_{-\nu-1}(R) , \quad (40)$$

where a_i^1, a_2^j, b_1^j and b_2^j are arbitrary constants and

$$A^{+} \equiv \sqrt{\frac{p - p_z}{p + p_z}} \quad , \qquad A^{-} \equiv \sqrt{\frac{p + p_z}{p - p_z}} \, , \tag{41}$$

$$B^{+} \equiv \frac{\omega - p}{m} \quad , \qquad B^{-} \equiv \frac{\omega + p}{m} \, , \tag{42}$$

where $p = \sqrt{\omega^2 - m^2}$ is the total momentum of the quark and $m \equiv h^- v$ is its mass. The argument of the above functions is defined as

$$R \equiv \sqrt{p^2 - p_z^2} \ r \ , \tag{43}$$

and the subscript is defined as

$$\nu \equiv j + 2q_R^- = j + 1 + 2q_L^- \,. \tag{44}$$

Among six unknown coefficients four can be removed by the junction condition of the internal solution (32), (33), (34), (35) and the external solution (37), (38), (39), (40). The other two should be determined by the boundary condition. In order to calculate the scattering cross section, we employ the incoming plain wave boundary condition at the infinity. The plain wave of the +helicity is written by

$$u_1^j|_{r\to\infty} = (-i)^j J_j(R) + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} + g_j \frac{e^{ip_\perp r}}{\sqrt{r}},$$
 (45)

$$u_2^j|_{r\to\infty} = i(-i)^j A^+ J_{j+1}(R) e^{i\theta} + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} A^+ e^{i\theta} - g_j \frac{e^{ip_\perp r}}{\sqrt{r}} A^- e^{i\theta} , \qquad (46)$$

$$w_1^j|_{r\to\infty} = (-i)^j B^+ J_j(R) e^{i\theta} + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^+ e^{i\theta} + g_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^- e^{i\theta} , \qquad (47)$$

$$w_2^j|_{r\to\infty} = i(-i)^j B^+ A^+ J_{j+1}(R) e^{2i\theta} + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^+ A^+ e^{2i\theta} - g_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^- A^- e^{2i\theta} , (48)$$

and the -helicity plain wave is written by

$$u_1^j|_{r\to\infty} = (-i)^j J_j(R) + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} + g_j \frac{e^{ip_\perp r}}{\sqrt{r}},$$
 (49)

$$u_2^j|_{r\to\infty} = (-i)^{j+1} A^- J_{j+1}(R) e^{i\theta} + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} A^+ e^{i\theta} - g_j \frac{e^{ip_\perp r}}{\sqrt{r}} A^- e^{i\theta} , \qquad (50)$$

$$w_1^j|_{r\to\infty} = (-i)^j B^- J_j(R) e^{i\theta} + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^+ e^{i\theta} + g_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^- e^{i\theta} , \qquad (51)$$

$$w_2^j|_{r\to\infty} = (-i)^{j+1} B^- A^- J_{j+1}(R) e^{2i\theta} + f_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^+ A^+ e^{2i\theta} - g_j \frac{e^{ip_\perp r}}{\sqrt{r}} B^- A^- e^{2i\theta} , (52)$$

where $p_{\perp} \equiv \sqrt{p^2 - p_z^2}$ is the quark momentum perpendicular to the string axis.

After a slight tedious calculation, the coefficients, f_j and g_j , can be obtained from which the scattering cross section per unit string length, $d\sigma_s/dl$, can be deduced by

$$\frac{d\sigma_s}{dld\theta} = \sum_j |f_j|^2 + \sum_j |g_j|^2 . {(53)}$$

Although the exact evaluation depends on D^+ , A^+ and so on, the important feature is that $d\sigma_s/dl$ is proportional to the inverse of p_{\perp} as

$$\frac{d\sigma_s}{dl} \propto \frac{1}{p_\perp} \ . \tag{54}$$

When $p_{\perp} \ll \delta_s^{-1}$ holds, the Z-charge of the quark appears as

$$\frac{d\sigma_s}{dl} \sim \frac{\sin^2\left(2q_R^-\pi\right)}{p_\perp} \ . \tag{55}$$

Otherwise, that is, $p_{\perp} \gtrsim \delta_s^{-1}$, the factor $\sin^2(2q_R^-\pi)$ disappears which is not mentioned in the references [20].

Now we return to the case of w^+ and u^+ . The spinors can be decomposed as

$$u^{+} = \sum_{j=-\infty}^{+\infty} \begin{pmatrix} \tilde{u}_{1}^{j}(r) \\ i\tilde{u}_{2}^{j}(r)e^{i\theta} \end{pmatrix} e^{i(j+1)\theta + ip_{z}z - i\omega t} , \qquad (56)$$

$$w^{+} = \sum_{j=-\infty}^{+\infty} \begin{pmatrix} \tilde{w}_{1}^{j}(r) \\ i\tilde{w}_{2}^{j}(r)e^{i\theta} \end{pmatrix} e^{ij\theta + ip_{z}z - i\omega t} , \qquad (57)$$

and the Z-charges of the upper quarks are

$$q_R^+ = -\frac{2}{3}\sin^2\theta_w , \qquad q_L^+ = \frac{1}{2} - \frac{1}{3}\sin^2\theta_w .$$
 (58)

Instead of the equation (44), the relation,

$$\nu' \equiv j + 1 + 2q_R^+ = j + 2q_L^+ \,, \tag{59}$$

holds so that the solutions for the upper quarks can be acquired by the similar way as the lower ones and the scattering cross section can be deduced. The result proves $d\sigma_s/dl$ has the same characteristic as the equation (54). The additional factor when $p_{\perp} \ll \delta_s^{-1}$ is altered to

$$\frac{d\sigma_s}{dl} \sim \frac{\sin^2\left(2q_L^+\pi\right)}{p_\perp} \ . \tag{60}$$

In the standard model where $\sin^2\theta_w \simeq 0.23$, $2q_R^- \simeq 0.15$ and $2q_L^+ \simeq 0.69$ so that the factors $\sin^2(2q_R^-\pi)$ or $\sin^2(2q_L^+\pi)$ are not so significant. Even if the non-standard value of $\sin^2\theta_w$ is employed, the essential feature that σ_s is proportional to $1/p_\perp$ which can be seen in the equation (54) is not altered. Of course, when $\sin^2(2q_R^-\pi) \ll 1$ and $\sin^2(2q_L^+\pi) \ll 1$ hold, the interaction of the string with the quarks is suppressed and the enhancement by small p_\perp might be overcome. Particularly when $\sin^2(2q_R^-\pi)$ and/or $\sin^2(2q_L^+\pi)$ equal exactly zero, the cross section has different dependence from the equations (55) and (60).

Such a situation can be realized within the standard model since the change of $\sin^2 \theta_w$ has the same effect as the variation of q_R^- or q_L^+ in the calculation of the particle scatter. The quark charge for the electroweak W-string is effectively a half-integer, that is, $\sin(2q_R^-\pi) = 0$ in the equation (55). Then we have to use N_{ν} other than $J_{-\nu}$ in the equation (37), (38), (39), (40) since J_{ν} and $J_{-\nu}$ are linearly dependent when ν is an integer. For the reason that an integer-order Neumann function has an logarithmic dependence, an extra factor comes out as

$$\frac{d\sigma_s}{dl} \sim \frac{1}{p_\perp} \frac{1}{\ln^2 \frac{p_\perp \delta_s}{2}} \,, \tag{61}$$

when $p \gg m$ besides $p_{\perp} \ll \delta_s^{-1}$. This factor is neither important since $\left(\ln \frac{p_{\perp} \delta_s}{2}\right)^{-2}$ becomes very small when $p_{\perp} \ll \delta_s^{-1}$. We are interested in the cases that σ_s should be enhanced.

Finally we take a brief look at the situation when the quark proceeds in parallel with the string axis. When $\omega = \sqrt{p_z^2 + m^2}$ as usual, the internal solution is written by the Bessel functions similarly to the formulae (32), (33), (34), (35) and the solution outside the string core damps in the power law manner as r^{ν} , $r^{\nu+1}$, $r^{-\nu}$, $r^{-\nu-1}$. Under the condition that $\omega = p_z$, the inner solution is expressed by the power law functions or constants and the outer one damps exponentially as e^{-mr} . It is natural that the smaller the momentum perpendicular to the string axis the larger the amplitude of the wave function of the particle inside the string since p_{\perp} supplies the energy which is converted to the mass, m, outside the string.

4 Conclusions

In the previous section, we have calculated the scattering cross section of the quarks from the electroweak strings. The result can be summarized as

$$\frac{d\sigma_s}{dl} \sim \frac{1}{p_\perp} \ . \tag{62}$$

The length along the string where the baryogenesis occurs sufficiently can be estimated to be at least $\sim \delta_s$ since only the region where the boundary between the false vacuum and the true vacuum is in question. Hence the quark scattering cross section should be no less than

$$\sigma_s \sim \frac{\delta_s}{p_\perp}$$
 (63)

As the next step we calculate the averaged value of $1/p_{\perp}$. If the homogeneous probability distribution of p_{\perp} is assumed then

$$\left\langle \frac{1}{p_{\perp}} \right\rangle = \frac{1}{p} \int_{\Lambda}^{p} \frac{1}{p_{\perp}} dp_{\perp} = \frac{1}{p} \ln \left(\frac{p}{\Lambda} \right) , \qquad (64)$$

where Λ is the infrared cutoff scale. Since the momentum of the quark should be the scale of the temperature at the electroweak phase transition, *i. e.*, the scale of the Higgs mass $\sim \delta_s^{-1}$,

$$\left\langle \frac{1}{p_{\perp}} \right\rangle \sim \delta_s \ln \left(\frac{\Lambda^{-1}}{\delta_s} \right) .$$
 (65)

Thus the enhance factor of σ_s relative to the geometrical cross section, δ_s^2 , is

$$\ln\left(\frac{\Lambda^{-1}}{\delta_s}\right) \sim 40 ,$$
(66)

when Λ is taken to be the inverse of the horizon scale at the electroweak symmetry restoration epoch, $t \sim 10^{14} \ {\rm GeV^{-1}}$. This is not a radical amplification. We, however, treat the quarks from all the direction equivalently. The most effective particle for the baryon number production is one which is running against the string end vertically. The damping of the wave function in the region outside the string core suggests that σ_s can be expected to be much larger.

Therefore the effective increase of the interaction area between strings and particles may partially cancel the smallness of the suppression factor (9). Such an enhancement is probable since the mass of the quarks vanishes in the string core where the electroweak symmetry is restored so that they prefer to reside inside the string. Even if the Kibble mechanism is valid and the suppression factor is not so small, the increase of the produced baryon asymmetry estimation generally means that the needed CP violation strength can be lower and the constraints on the model should be relaxed.

The estimation in this paper is only rough one. The precise value of the interaction cross section should depend on the basic process of the baryon generation. Further investigation might reveal the usefulness of the baryogenesis by the electroweak strings.

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References

- [1] E. W. Kolb and M. S. Turner, *The Early Universe*, (Addison-Wesley Publishing Company, Redwood City, 1990) p.157 and references therein.
- [2] T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive and H.-S. Kang, Astrophys. J. 376, 51 (1991).
- [3] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 43, 27 (1993).
- [4] A. D. Sakharov, JETP Lett. 5, 24 (1967).
- [5] G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976).
 M. S. Manton, Phys. Rev. D28, 2019 (1983).
- [6] G. R. Farrar and M. Shaposhnikov, Phys. Rev. **D50**, 774 (1994).
- [7] T. Shiromizu, M. Morikawa and J. Yokoyama, Prog. Theor. Phys. 94, 795 (1995).
- [8] R. H. Brandenberger and A.-C. Davis, Phys. Lett. **B308**, 79 (1993).
- [9] T. W. B. Kibble, J. Phys. **A9**, 1387 (1976).
- [10] T. W. B. Kibble, Phys. Rep. **67**, 183 (1980).
 - A. Vilenkin, Phys. Rep. 121, 263 (1985).
 - A. Vilenkin and E. P. S. Shellard, Cosmic Strings and Other Topological Defects, (Cambridge University Press, New York, 1994).
- [11] Y. Nambu, Nucl. Phys. **B130**, 505 (1977).
 - V. Soni, Phys. Lett. **B93**, 101 (1980).
 - A. Sugamoto, Phys. Lett. **B127**, 75 (1983).
 - T. Vachaspati, Phys. Rev. Lett. 68, 1977 (1992).

- T. Vachaspati, Nucl. Phys. **B397**, 648 (1993).
- G. Dvali and G. Senjanović, Phys. Rev. Lett. 71, 2376 (1993).
- G. Dvali and G. Senjanović, Phys. Lett. **B331**, 63 (1994).
- [12] T. Vachaspati, Phys. Rev. Lett. 73, 373 (1994).
 M. Sato and S. Yahikozawa, Nucl. Phys. 436, 100 (1995).
- [13] M. Barriola, Phys. Rev. **D51**, R300 (1995).
- [14] V. Soni, preprint NPL-TH-94-1, hep-ph/9505386 (1995).
- [15] M. James, L. Perivolaropoulos and T. Vachaspati, Phys. Rev. **D46**, R5232 (1992).
 W. B. Perkins, Phys. Rev. **D47**, R5224 (1993).
 - M. James, L. Perivolaropoulos and T. Vachaspati, Nucl. Phys. **B395**, 534 (1993).
 - T. Vachaspati and R. Watkins, Phys. Lett. **B318**, 163 (1993).
 - A. Achúcarro, R. Gregory, J. A. Harvey and K. Kuijken, Phys. Rev. Lett. **72**, 3646 (1994).
 - M. A. Earnshaw and W. B. Perkins, Phys. Lett. **B328**, 337 (1994).
 - J. M. Moreno, D. H. Oaknin and M. Quirós, Phys. Lett. **B347**, 332 (1995).
 - G. Bimonte and G. Lozano, Phys. Lett. **B348**, 457 (1995).
 - S. G. Naculich, Phys. Rev. Lett. **75**, 998 (1995).
 - L. Masperi and A. Mégevand, preprint hep-ph/9410211 (1996) to be published in Zeitschrift fur Physik C.
- [16] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973).
- [17] T. Vachaspati and M. Barriola, Phys. Rev. Lett. 69, 1867 (1992).
 M. Barriola, T. Vachaspati and M. Bucher, Phys. Rev. D50, 2819 (1994).
- [18] R. H. Brandenberger, A.-C. Davis and M. Trodden, Phys. Lett. **B335**, 123 (1994).
- [19] M. Trodden, A.-C. Davis and R. H. Brandenberger, Phys. Lett. **B349**, 131 (1995).
- [20] N. Ganoulis, Phys. Lett. **B298**, 63 (1993).
 A. C. Davis, A. P. Martin and N. Ganoulis, Nucl. Phys. **B419** 323 (1994).
 H.-K. Lo, Phys. Rev. **D51**, 802 (1995).